PRINT your name and student ID: ____________________

Section 2: Free-form Problems (94+15 points)

5. DFT for Impulse Noise Cancellation

We explore the relationship between error-control codes and DFT in the problem. Consider a discrete-time sequence of length n, $\vec{x} \in \mathbb{R}^n$, as the signal we want to transmit. We further assume that the DFT coefficients of the signal are zero (0) at certain locations, where the locations are *known* to us. After transmission over a noisy channel, an additive impulse noise of *unknown* magnitude is added to one of the *unknown* locations, k, i.e., if the received signal is denoted as $\vec{y}$, then $y(t) = x(t)$ for $t \neq k$, and $y(k) = x(k) + a$, where a is the unknown magnitude of the additive impulse noise, and we also don't know k. The question is: *Can we recover the original signal by revealing the impulse noise, i.e., identify a and k.*

Let's make it concrete. Assume the signal we want to transmit is of length 7, i.e., $n = 7$, and we know that its DFT coefficients are 0 at locations $p = 3$ and 4. The original signal $\vec{x}$ and its DFT coefficients $\vec{X}$ are shown below (note that the $X(p) = 0$ for $p = 3, 4$).

t	0	1	2	3	4	5	6
$x(t)$	0.7559	0.1164	0.2431	1.9083	1.9083	0.2431	0.1164

p	0	1	2	**3**	**4**	5	6
$X(p)$	2	-1	1	**0**	**0**	1	-1

Now we will ask a series of questions to see if you really understand DFT.

(a) How do you get $X(p)$ from $\vec{x}$? How to convert back to $x(t)$ from $\vec{X}$? Here you can use $x(t)$ and $X(p)$ without plugging in the actual values, and assume $n = 7$ as in the above table.

(b) To achieve error correction, we assume that the DFT coefficients, $\vec{X}$, are zero at locations 3 and 4, i.e., $X(3) = X(4) = 0$. Write out this constraint in the form of two equations using values of the original signal $\vec{x}$. Here you can assume $n = 7$ (note the signal in the above table is an instance that should satisfy this general constraint).

(c) After transimission, the received signal $\vec{y}$ is listed as below. Since you are provided with the original signal, you can see that an impulse of magnitude 1 is added to location 2, i.e., $y(2) = x(2) + 1$, and $y(t) = x(t)$ for $t \neq 2$.

t	0	1	2	3	4	5	6
$y(t)$	0.7559	0.1164	**1.2431**	1.9083	1.9083	0.2431	0.1164

In practice, however, you won't have access to the original signal, so you won't know where the impulse is added and what the magnitude is. Can we identify these crucial quantities? YES! Now, please first obtain the DFT of $\vec{y}$ at locations 3 and 4 based on the signal values of $\vec{y}$, i.e., what is $Y(3)$ and $Y(4)$? (Hint: you don't need to do it the hard way ;)

(d) In the above question, the impulse noise of magnitude $a = 1$ is added to time $k = 2$. For the general case, with $k \in \{0, 1, ..., 6\}$, what will be $Y(3)$ and $Y(4)$? Your answer should depend on a and k.

(e) Given that you have correctly answered the above question, you should readily see how to get a and k from $Y(3)$ and $Y(4)$. Please write out the relationship. (You may take the logarithm of a complex number, $\ln ae^{i\theta} = i\theta + \ln a$.)

Great! You have just gone through the simplest case for error correction based on DFT.

Note: this problem is made by Ming Jin for EE16B Midterm Exam, 2016